1 The dynamics of the rate of profit

Let \( S(t) \) denote the annual surplus value (measured in person-hours per annum—or just persons, since the hours and the “per annum” effectively cancel out) and let \( K(t) \) denote the capital stock (measured in person-hours). We define the rate of profit (with dimension time\(^{-1}\)) as

\[
R(t) = \frac{S(t)}{K(t)} \tag{1}
\]

Consider the proportional rate of change of the rate of profit:

\[
\frac{\dot{R}(t)}{R(t)} = \frac{\dot{S}(t)}{S(t)} - \frac{\dot{K}(t)}{K(t)} \tag{2}
\]

We first work on \( \dot{S}(t)/S(t) \). Let us decompose \( S(t) \) as the profit share in value-added (\( \pi \)) times the total value-added, \( L(t) \). (By “value-added” we really mean the total labour performed per unit time.) For the moment we will assume that \( \pi \) is fixed, while \( L(t) \) grows at a constant proportional rate \( n \). That is,

\[
S(t) = \pi L(t) \quad 0 < \pi < 1
\]

and

\[
\frac{\dot{S}(t)}{S(t)} = \frac{\dot{L}(t)}{L(t)} \equiv n \tag{3}
\]

We now work on \( \dot{K}(t)/K(t) \), the proportional rate of expansion of capital stock. We will at first assume that net investment, \( \dot{K}(t) \), is a constant fraction, \( 0 < i < 1 \), of surplus value:

\[
\frac{\dot{K}(t)}{K(t)} = \frac{iS(t)}{K(t)} = \frac{i\pi L(t)}{K(t)} \tag{4}
\]

Under the above-mentioned assumptions, the proportional rate of change of the rate of profit can be written as

\[
\frac{\dot{R}(t)}{R(t)} = \frac{\dot{S}(t)}{S(t)} - \frac{\dot{K}(t)}{K(t)} = n - i \frac{S(t)}{K(t)} = n - iR(t) \tag{5}
\]

The rate of profit is unchanging if and only if

\[
n - iR(t) = 0
\]

or

\[
R(t) = R^* = \frac{n}{i} \tag{6}
\]

If the investment fraction, \( i \), is 100 percent, the equilibrium rate of profit equals \( n \); if the investment fraction is 50 percent, the equilibrium rate of profit is \( 2 \times n \), and so on.
Stability of equilibrium

Is the equilibrium expressed in equation (6) stable? Suppose we start from some $R(0)$ which differs from $R^\star$. Since the coefficient on $R(t)$ in (5) is negative, it is easily seen that the rate of profit must converge on the equilibrium. If $R(0) > R^\star$ the rate of profit falls, and if $R(0) < R^\star$ the rate of profit rises, over time.

For example, if $n = 0.05$, $i = 0.80$ and $R(0) = 0.20$, we have the rate of profit changing at a rate of $0.05 - 0.80 \times 0.20 = -0.11$, or decaying by 11 percent per year. As $R^\star$ is approached, the rate of decay itself declines.

The effect of change in the profit share

For any given profit share, we can use (4) and (5) to get

$$\frac{\dot{R}(t)}{R(t)} = n - i\pi \frac{L(t)}{K(t)}$$

This tells us that an increase in $\pi$ will reduce $\dot{R}/R(t)$. In other words, it will reduce the rate at which $R^\star$ is approached when $R(t)$ differs from $R^\star$. On the other hand, the value of $\pi$ does not enter the relationship (6): the equilibrium rate of profit is invariant with respect to the profit share.

Note: From the relationship $S(t) = \pi L(t)$, it might appear that we could write a more general version of (3) as

$$\frac{\dot{S}(t)}{S(t)} = \frac{\dot{\pi}(t)}{\pi(t)} + \frac{\dot{L}(t)}{L(t)} = \lambda + n$$

and re-trace the dynamics on the assumption that $\lambda > 0$. However, this would not be valid. The profit share, $\pi$, has an upper bound of 1.0 so we cannot postulate a constant positive value for $\dot{\pi}/\pi(t)$. What we can do is express $\pi$ as $1 - w$, where $w$ is the wage share in value added, and examine the effect of an ongoing decay in $w$:

$$w_t = (1 - g)w_{t-1} \quad 0 < g < 1$$

For example $g = 0.05$ would give an annual shrinkage of 5 percent in the wage share, with the profit share therefore tending to 1.0. This is shown in [Figure 3 in the paper]. As stated above, the effect is to delay the approach to equilibrium without altering the equilibrium itself.

2 The capital–labour ratio

Since $R(t) = \pi L(t)/K(t)$, the equilibrium condition for the rate of profit (6) implies a corresponding equilibrium condition for the capital-labour ratio. At $R(t) = R^\star$

$$\frac{\pi L(t)}{K(t)} = \frac{n}{i}$$
and so
\[
\frac{K(t)}{L(t)} = \frac{\pi}{n}
\]

For example, with \(n = 0.05, \pi = 0.5\) and \(i = 0.80\) the implied equilibrium value for \(K/L\) is 8.0.

When the rate of profit stands above its equilibrium value, the capital–labour ratio stands below equilibrium. For example, given \(\pi = 0.5\) and \(i = 0.80\) as above, but with a profit rate of \(R(t) = 0.20 > n/i = 0.0625\), the implied value of \(K/L\) is found from \(R(t) = \pi L(t)/K(t)\) to be \(\pi/R(t) = 2.5\).

As the actual rate of profit declines towards equilibrium, therefore, there is an implied progressive increase in the capital–labour ratio. Question: how exactly do we conceptualize this? For example, as involving technological change, or as moving along a production function? Does it matter?