

## 1 The dynamics of the rate of profit

Let  $S(t)$  denote the annual surplus value (measured in person-hours per annum—or just persons, since the hours and the “per annum” effectively cancel out) and let  $K(t)$  denote the capital stock (measured in person-hours). We define the rate of profit (with dimension  $\text{time}^{-1}$ ) as

$$R(t) = S(t)/K(t) \tag{1}$$

Consider the proportional rate of change of the rate of profit:

$$\frac{\dot{R}(t)}{R(t)} = \frac{\dot{S}(t)}{S(t)} - \frac{\dot{K}(t)}{K(t)} \tag{2}$$

We first work on  $\dot{S}(t)/S(t)$ . Let us decompose  $S(t)$  as the profit share in value-added ( $\pi$ ) times the total value-added,  $L(t)$ . (By “value-added” we really mean the total labour performed per unit time.) For the moment we will assume that  $\pi$  is fixed, while  $L(t)$  grows at a constant proportional rate  $n$ . That is,

$$S(t) = \pi L(t) \quad 0 < \pi < 1$$

and

$$\frac{\dot{S}(t)}{S(t)} = \frac{\dot{L}(t)}{L(t)} \equiv n \tag{3}$$

We now work on  $\dot{K}(t)/K(t)$ , the proportional rate of expansion of capital stock. We will at first assume that net investment,  $\dot{K}(t)$ , is a constant fraction,  $0 < i < 1$ , of surplus value:

$$\frac{\dot{K}(t)}{K(t)} = \frac{iS(t)}{K(t)} = \frac{i\pi L(t)}{K(t)} \tag{4}$$

Under the above-mentioned assumptions, the proportional rate of change of the rate of profit can be written as

$$\frac{\dot{R}(t)}{R(t)} = \frac{\dot{S}(t)}{S(t)} - \frac{\dot{K}(t)}{K(t)} = n - i \frac{S(t)}{K(t)} = n - iR(t) \tag{5}$$

The rate of profit is unchanging if and only if

$$n - iR(t) = 0$$

or

$$R(t) = R^* = \frac{n}{i} \tag{6}$$

If the investment fraction,  $i$ , is 100 percent, the equilibrium rate of profit equals  $n$ ; if the investment fraction is 50 percent, the equilibrium rate of profit is  $2 \times n$ , and so on.

### *Stability of equilibrium*

Is the equilibrium expressed in equation (6) stable? Suppose we start from some  $R(0)$  which differs from  $R^*$ . Since the coefficient on  $R(t)$  in (5) is negative, it is easily seen that the rate of profit must converge on the equilibrium. If  $R(0) > R^*$  the rate of profit falls, and if  $R(0) < R^*$  the rate of profit rises, over time.

For example, if  $n = 0.05$ ,  $i = 0.80$  and  $R(0) = 0.20$ , we have the rate of profit changing at a rate of  $0.05 - 0.80 \times 0.20 = -0.11$ , or decaying by 11 percent per year. As  $R^*$  is approached, the rate of decay itself declines.

### *The effect of change in the profit share*

For any given profit share, we can use (4) and (5) to get

$$\frac{\dot{R}(t)}{R(t)} = n - i\pi \frac{L(t)}{K(t)}$$

This tells us that an increase in  $\pi$  will reduce  $\dot{R}/R(t)$ . In other words, it will reduce the rate at which  $R^*$  is approached when  $R(t)$  differs from  $R^*$ . On the other hand, the value of  $\pi$  does not enter the relationship (6): the equilibrium rate of profit is invariant with respect to the profit share.

*Note:* From the relationship  $S(t) = \pi L(t)$ , it might appear that we could write a more general version of (3) as

$$\frac{\dot{S}(t)}{S(t)} = \frac{\dot{\pi}(t)}{\pi(t)} + \frac{\dot{L}(t)}{L(t)} = \lambda + n$$

and re-trace the dynamics on the assumption that  $\lambda > 0$ . However, this would not be valid. The profit share,  $\pi$ , has an upper bound of 1.0 so we cannot postulate a constant positive value for  $\dot{\pi}/\pi(t)$ . What we can do is express  $\pi$  as  $1 - w$ , where  $w$  is the wage share in value added, and examine the effect of an ongoing decay in  $w$ :

$$w_t = (1 - g)w_{t-1} \quad 0 < g < 1$$

For example  $g = 0.05$  would give an annual shrinkage of 5 percent in the wage share, with the profit share therefore tending to 1.0. This is shown in [Figure 3 in the paper]. As stated above, the effect is to delay the approach to equilibrium without altering the equilibrium itself.

## **2 The capital–labour ratio**

Since  $R(t) = \pi L(t)/K(t)$ , the equilibrium condition for the rate of profit (6) implies a corresponding equilibrium condition for the capital-labour ratio. At  $R(t) = R^*$

$$\frac{\pi L(t)}{K(t)} = \frac{n}{i}$$

and so

$$\frac{K(t)}{L(t)} = \frac{\pi i}{n}$$

For example, with  $n = 0.05$ ,  $\pi = 0.5$  and  $i = 0.80$  the implied equilibrium value for  $K/L$  is 8.0.

When the rate of profit stands above its equilibrium value, the capital-labour ratio stands below equilibrium. For example, given  $\pi = 0.5$  and  $i = 0.80$  as above, but with a profit rate of  $R(t) = 0.20 > n/i = 0.0625$ , the implied value of  $K/L$  is found from  $R(t) = \pi L(t)/K(t)$  to be  $\pi/R(t) = 2.5$ .

As the actual rate of profit declines towards equilibrium, therefore, there is an implied progressive increase in the capital-labour ratio. Question: how exactly do we conceptualize this? For example, as involving technological change, or as moving along a production function? Does it matter?